

# MESON SPECTRAL FUNCTIONS @ NON-ZERO MOMENTUM

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# HOT QCD: SPECTRAL FUNCTIONS

## MOTIVATION

deconfined phase: quarks and gluons

interesting observables

quark-gluon plasma at  $T_c \lesssim T \lesssim 3T_c$ :

- survival of bound states: charmonium, ...
- dilepton and photon production:  $\Pi^{\mu\nu} \sim \langle j^\mu j^\nu \rangle$
- transport coefficients: viscosity, conductivity, ...

observables best expressed in terms of spectral functions  
(particular *real-time* correlation functions)

# HOT QCD: SPECTRAL FUNCTIONS

## MOTIVATION

$$G_E(\tau, \mathbf{p}) = \int_0^\infty \frac{d\omega}{2\pi} K(\tau, \omega) \rho(\omega, \mathbf{p})$$

$$K(\tau, \omega) = \frac{\cosh[\omega(\tau - 1/2T)]}{\sinh(\omega/2T)}$$

some results at high  $T$ :

- bound states: charmonium survives in the QGP until  $T \lesssim 2T_c$  (Hatsuda & Asakawa, Bielefeld)
- dileptons:  $\rho_V$  very small (zero?) when  $\omega \lesssim 2T$  ? (Bielefeld 2001)
- conductivity: anticipated ‘bump’ at small energies (S. Gupta 2003)

# OVERVIEW

- motivation from transport and hydrodynamics:  
small energy behaviour of spectral functions
- lattice QCD: meson correlators and spectral functions  
at non-zero momentum
  - analytical results: lattice artefacts
  - numerical results: Maximum Entropy Method  
(preliminary)
- summary

# MESON SPECTRAL FUNCTIONS

SMALLISH ENERGIES

- transport coefficients from the lattice difficult:  $\omega \rightarrow 0$   
G.A. & J.M. MARTINEZ RESCO (2002)
- still focus on soft energies  $\omega \lesssim T$

meson spectral functions at non-zero momentum

- also non-trivial and interesting:
  - lightcone at  $\omega = p$
  - dilepton production
- but easier (presumably, hopefully, ...)

# MESON SPECTRAL FUNCTIONS

AT NON-ZERO MOMENTUM

meson spectral functions:

$$\rho_H(t, \mathbf{x}) = \langle [J_H(t, \mathbf{x}), J_H^\dagger(0, \mathbf{0})] \rangle$$

euclidean correlators:

$$G_H(\tau, \mathbf{x}) = \langle J_H(\tau, \mathbf{x}) J_H^\dagger(0, \mathbf{0}) \rangle$$

currents:

$$J_H(\tau, \mathbf{x}) = \bar{q}(\tau, \mathbf{x}) \Gamma_H q(\tau, \mathbf{x}) \quad \text{with} \quad \Gamma_H = \{\mathbb{1}, \gamma_5, \gamma^\mu, \gamma^\mu \gamma_5\}$$

expectation:

- below  $T_c$ : mesons moving in the heatbath  
simple sharply peaked quasiparticle spectral functions  
at  $\omega = \sqrt{p^2 + m^2}$  (+ subdominant continuum)
- above  $T_c$ : ‘free’ quarks, only continuum contribution

# MESON SPECTRAL FUNCTIONS

G.A. AND J.M. MARTÍNEZ RESCO, NPB (2005)

meson spectral functions:

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study analytically at infinite temperature (one loop)

continuum: analytical results  $\Leftrightarrow$  lattice: simple sums

lattice artefacts

- lightcone:  $\omega = p$
- below the lightcone:  $\omega^2 < p^2$ , Landau damping
- above threshold:  $\omega^2 > p^2 + 4m^2$

# CONTINUUM

COMPLETE ANALYTICAL RESULT (DON'T READ THIS!)

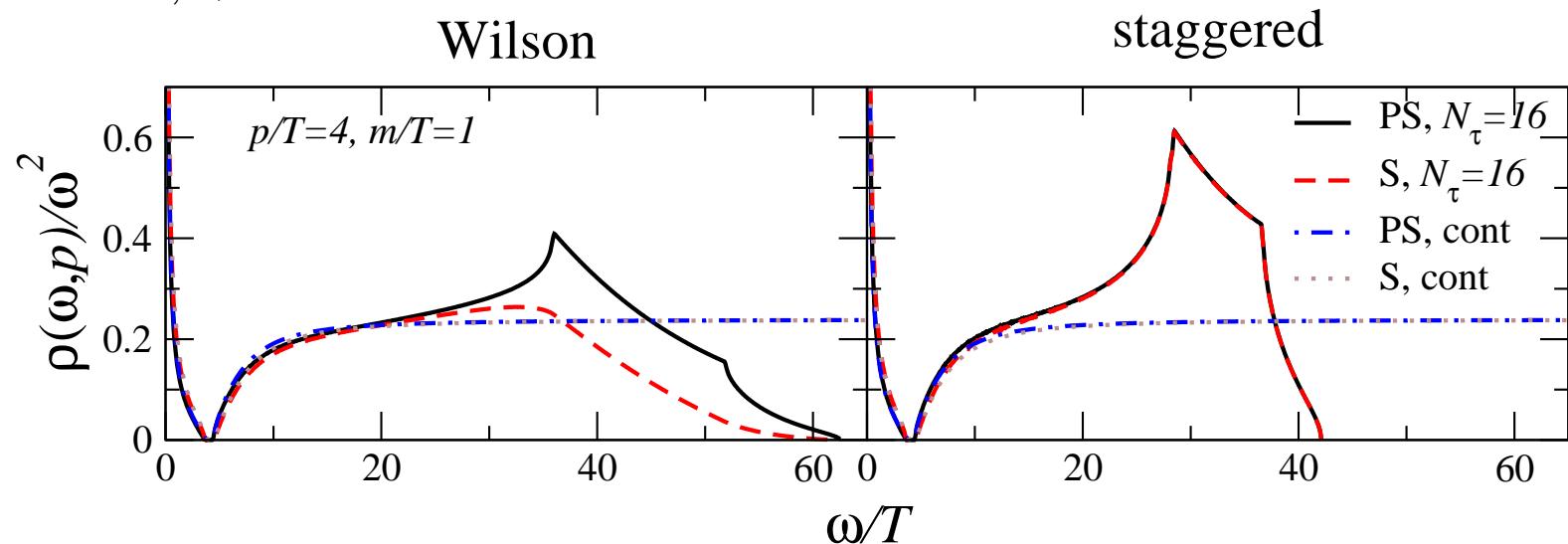
$$\begin{aligned}
\rho_H(\omega, \mathbf{p}) = & \Theta(s - 4m^2) \frac{N_c T^2}{\pi} \left\{ \right. \\
& \frac{\beta(P)}{24T^2} \left[ (3\omega^2 - p^2\beta^2(P)) a_H^{(1)} + (3p^2 - (3\omega^2 - 2p^2)\beta^2(P)) a_H^{(2)} - 12m^2 a_H^{(3)} \right] \\
& + \frac{1}{4pT} \left[ (\omega^2 - p^2\beta^2(P)) a_H^{(1)} + (p^2 - \omega^2\beta^2(P)) a_H^{(2)} - 4m^2 a_H^{(3)} \right] \ln \frac{1 + e^{-\bar{p}_+/T}}{1 + e^{-\bar{p}_-/T}} \\
& + \left( a_H^{(1)} + a_H^{(2)} \right) \left( \beta(P) \left[ \text{Li}_2(-e^{-\bar{p}_+/T}) + \text{Li}_2(-e^{-\bar{p}_-/T}) \right] \right. \\
& \quad \left. + \frac{2T}{p} \left[ \text{Li}_3(-e^{-\bar{p}_+/T}) - \text{Li}_3(-e^{-\bar{p}_-/T}) \right] \right) \left. \right\} \\
& + \Theta(-s) \frac{N_c T^2}{\pi} \left\{ \right. \\
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\end{aligned}$$

with  $\bar{p}_{\pm} = \frac{1}{2} [\omega \pm p\beta(P)]$ ,  $\beta(P) = \sqrt{1 - 4m^2/s}$ ,  $s = \omega^2 - p^2$

# SCALAR AND PSEUDOSCALAR

- large  $\omega$ :  $\rho_{PS,S}(\omega, p) \sim \omega^2$ , dimensional analysis
- small  $\omega$ :  $\rho_{PS,S}(\omega, p) \sim \omega$ , odd

show  $\rho_{PS,S}/\omega^2$ :

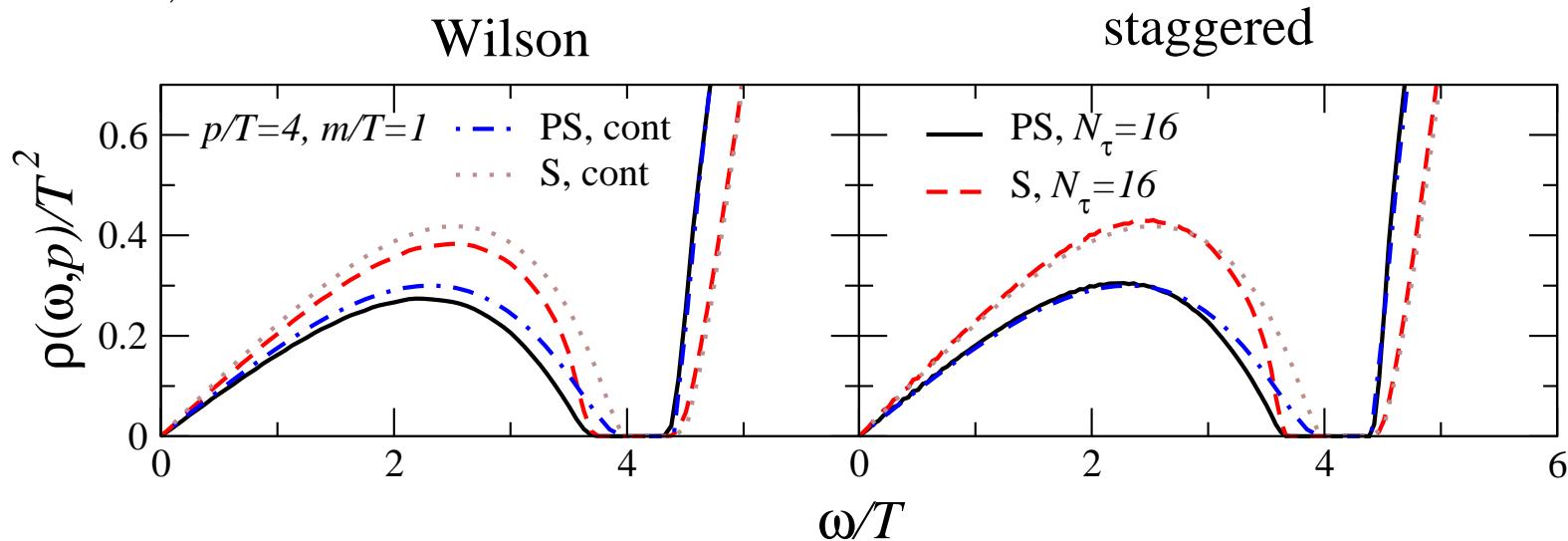


- $\omega \gg m$ : scalar and pseudoscalar degenerate for continuum and staggered, broken for Wilson fermions
- lightcone
- non-trivial low energy region

# SCALAR AND PSEUDOSCALAR

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- small  $\omega$ :  $\rho_{\text{PS,S}}(\omega, \mathbf{p}) \sim \omega$ , odd

show  $\rho_{\text{PS,S}}/T^2$ :



- $\omega \sim m$ : scalar and pseudoscalar non-degenerate
- lightcone affected by lattice artefacts
- below the lightcone: Landau damping

# QUENCHED QCD WITH STAGGERED FERMIONS

PRELIMINARY RESULTS, IN PROGRESS

quenched QCD:

ensembles below and above  $T_c$  with  $N_\tau = 24$  fixed  
(for MEM purposes)

- below  $T_c$ :

$48^3 \times 24$ ,  $\beta = 6.5$ ,  $a^{-1} \sim 4 \text{ GeV}$ ,  $T \sim 160 \text{ MeV}$

- above  $T_c$ :

$64^3 \times 24$ ,  $\beta = 7.192$ ,  $a^{-1} \sim 10 \text{ GeV}$ ,  $T \sim 420 \text{ MeV}$

100 propagators analyzed

staggered quarks:  $ma = 0.01, 0.05, 0.125$   
 $m/T = 0.24, 1.2, 3$

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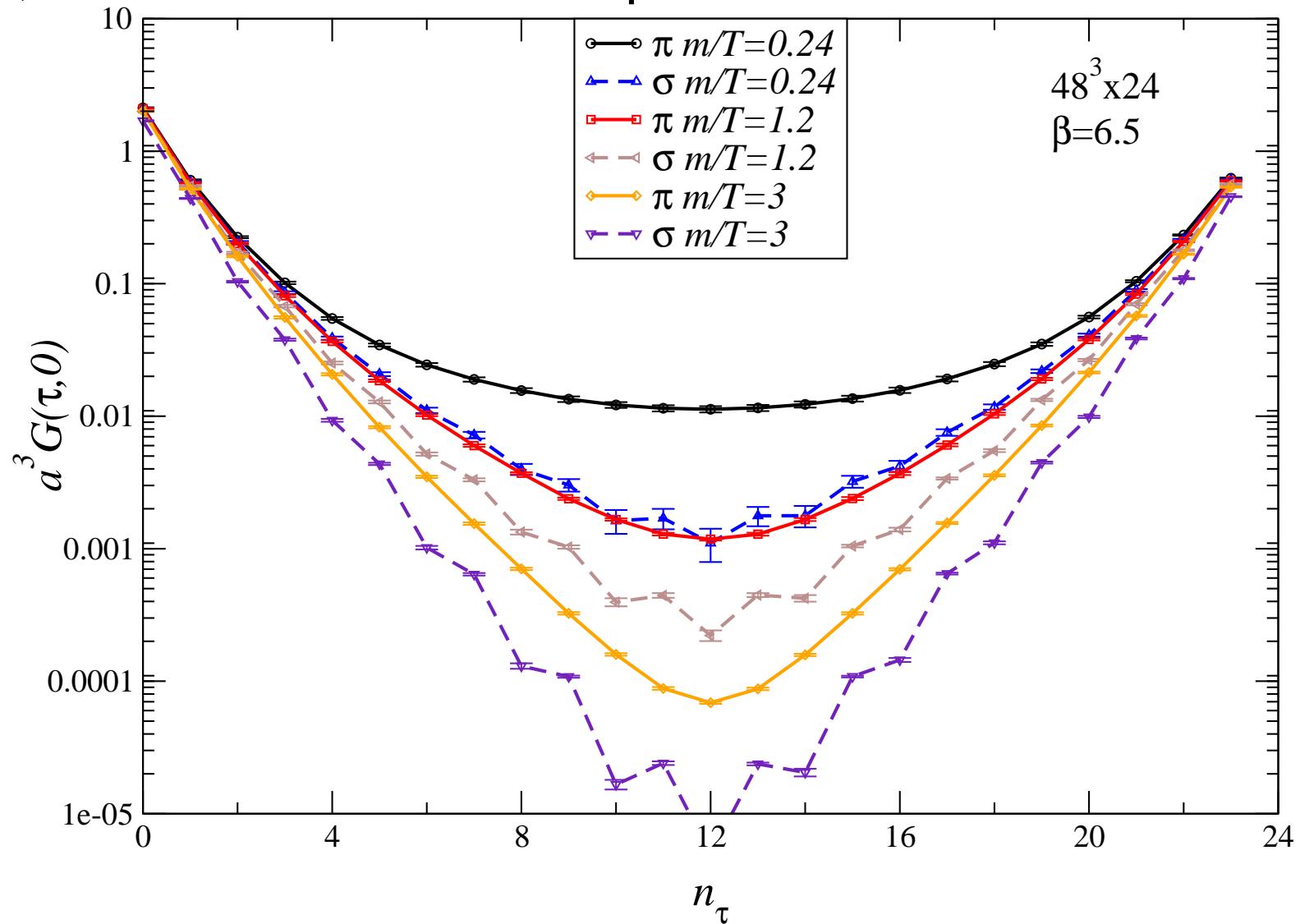
staggered quarks:  $ma = 0.01, 0.05, 0.125$   
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- chiral symmetry breaking/restoration:  
pseudoscalar vs. scalar for light quarks

# QUENCHED QCD WITH STAGGERED FERMIONS

PRELIMINARY RESULTS, IN PROGRESS

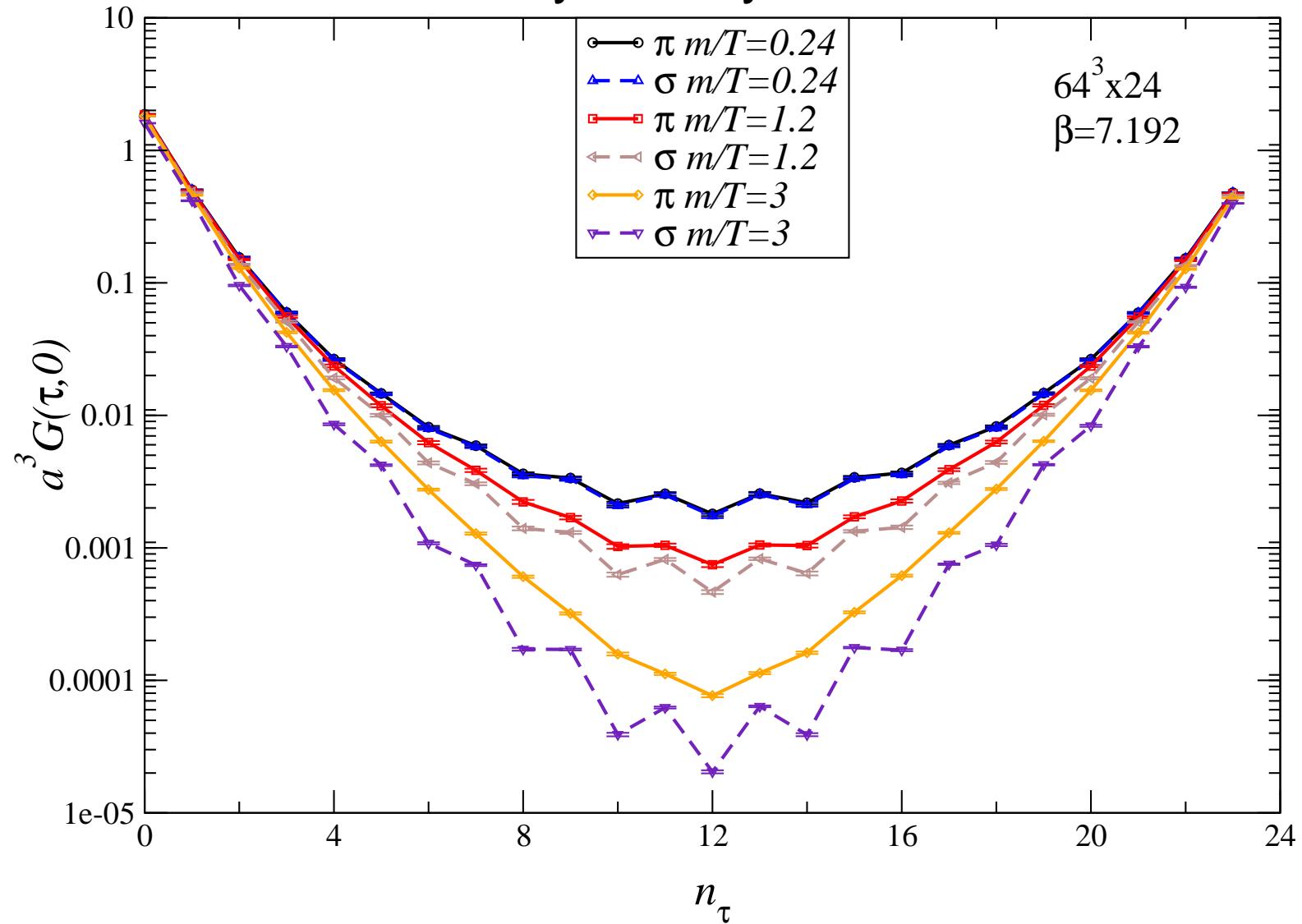
cold,  $T \approx 160$  MeV: confined phase



# QUENCHED QCD WITH STAGGERED FERMIONS

PRELIMINARY RESULTS, IN PROGRESS

hot,  $T \approx 420$  MeV: chiral symmetry restoration



# NON-ZERO MOMENTUM

WITH TWISTED BOUNDARY CONDITIONS

- momenta on the lattice  $\vec{p} = \frac{2\pi}{L}\vec{n}$

hydrodynamic regime: many momenta with  $p \lesssim T$

- large volume, large ratio  $N_\sigma/N_\tau$ , large  $N_\tau$ ?

twisted boundary conditions

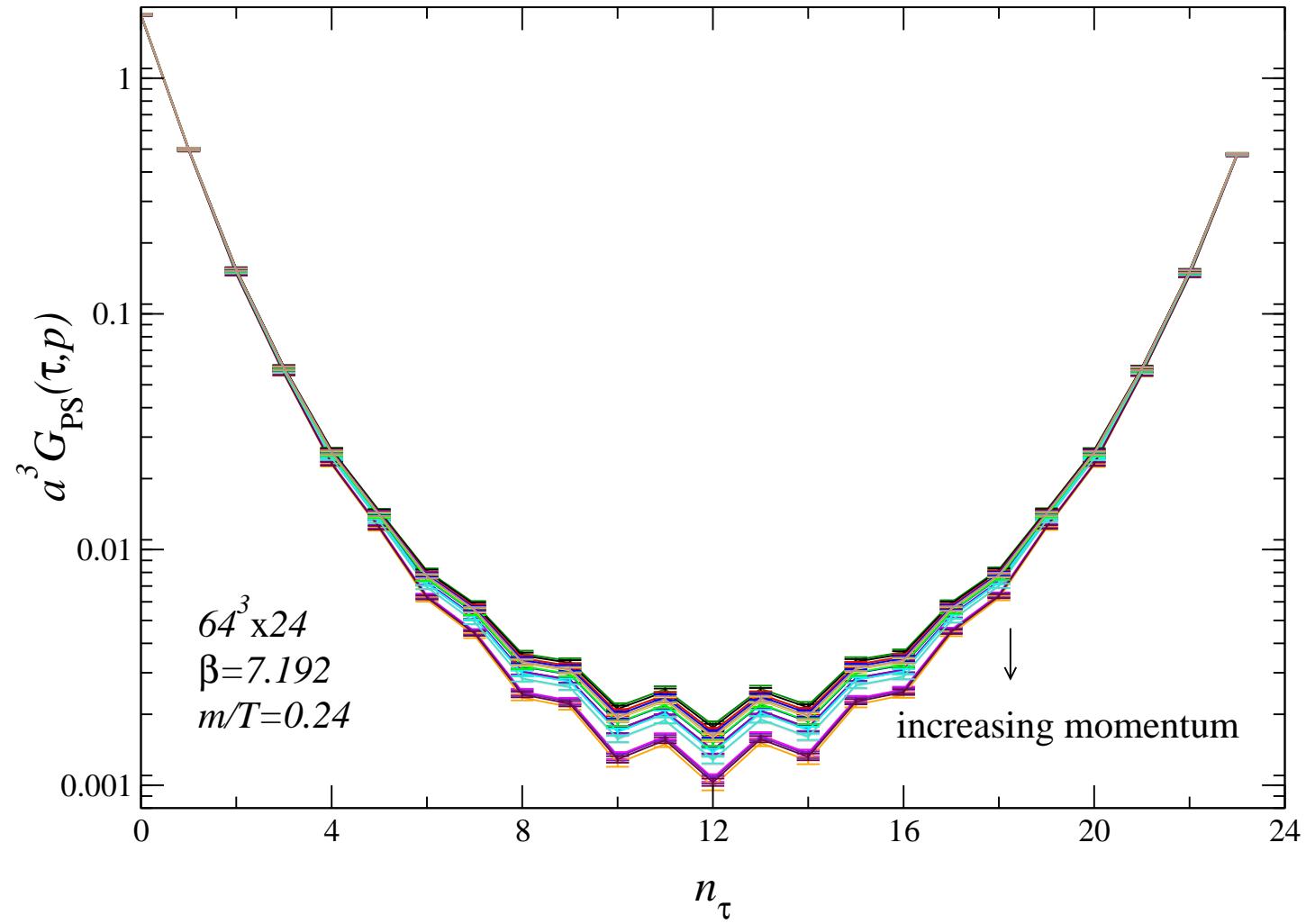
(long history, Flynn et.al. 2005)

- quark field:  $\psi(x_i + L) = e^{i\theta_i} \psi(x_i)$
- meson: two twist angles  $\vec{\theta}_1, \vec{\theta}_2$
- meson momentum:  $\vec{p} = \frac{2\pi}{L}\vec{n} - \frac{\vec{\theta}_1 - \vec{\theta}_2}{L}$
- $\sim 20$  momenta in the range  $0 < p/T < 5$

# HIGH TEMPERATURE QUENCHED QCD

SMALL (BARE) QUARK MASS,  $m/T = 0.24$

hot,  $T \approx 420$  MeV

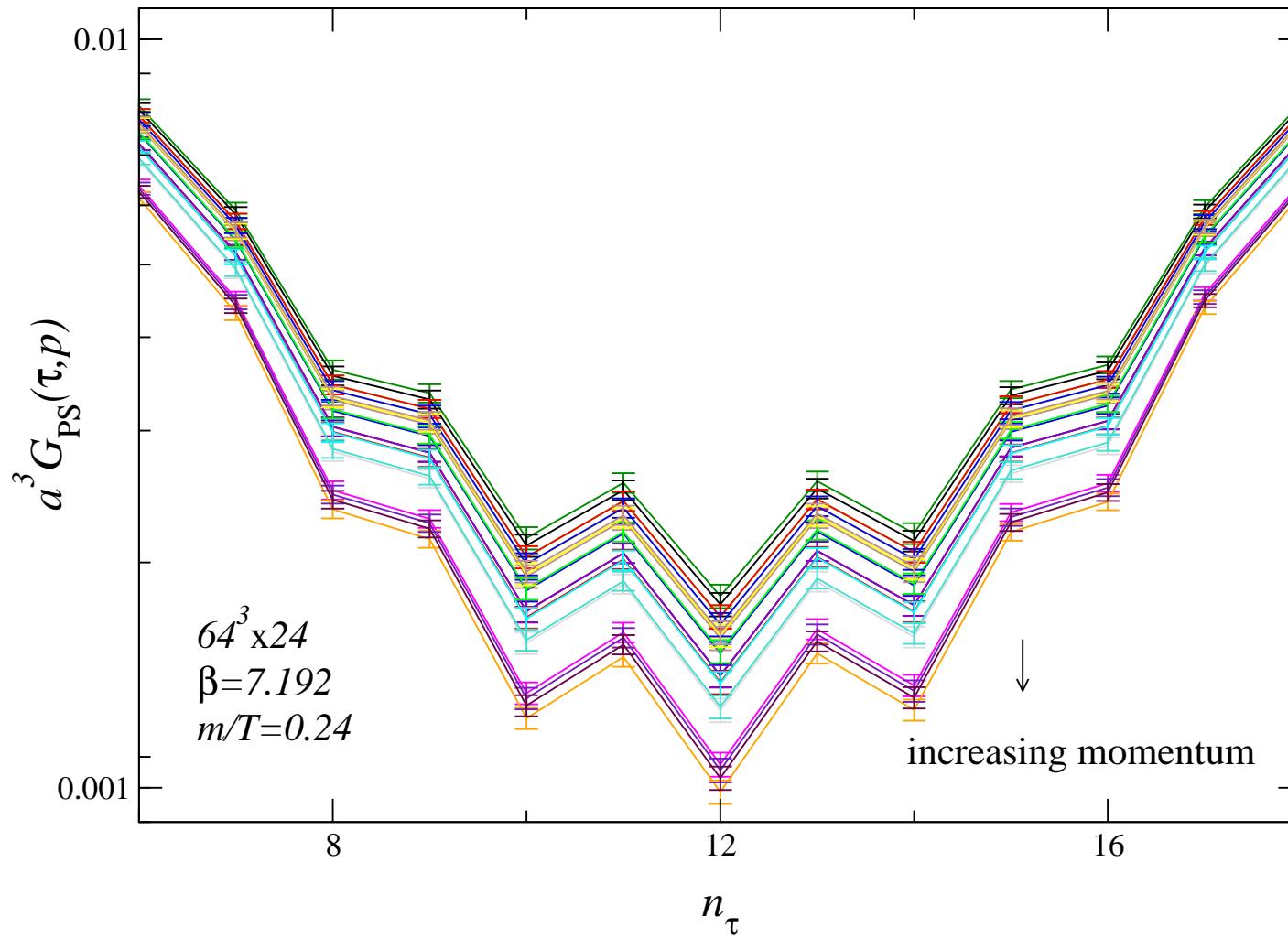


pseudoscalar:  $\sim 20$  independent momenta

# HIGH TEMPERATURE QUENCHED QCD

SMALL (BARE) QUARK MASS,  $m/T = 0.24$

hot,  $T \approx 420$  MeV

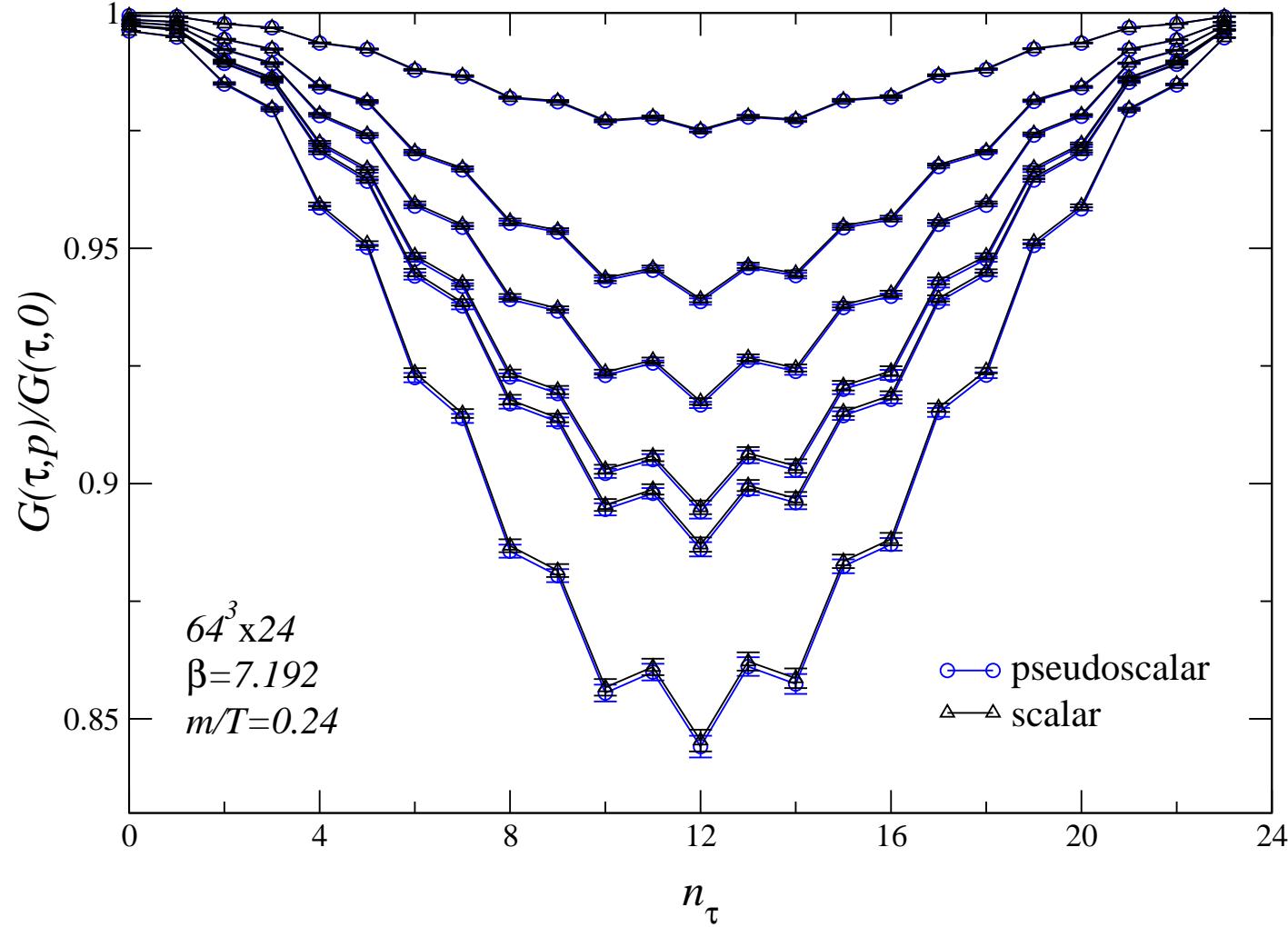


zoom in: small but non-zero effect, correctly ordered

# HIGH TEMPERATURE QUENCHED QCD

SMALL (BARE) QUARK MASS,  $m/T = 0.24$

hot,  $T \approx 420$  MeV

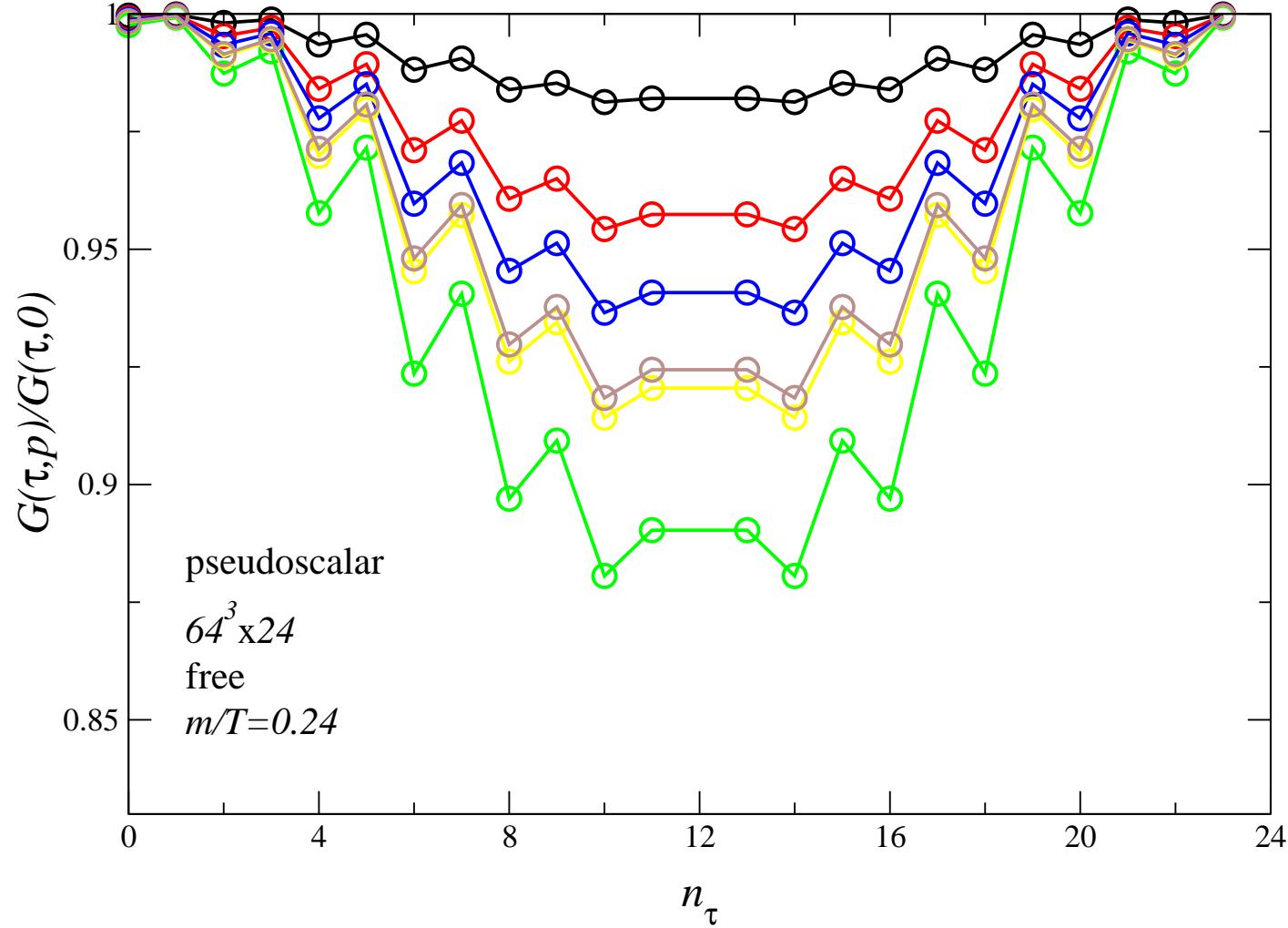


chiral symmetry restoration at non-zero momentum

# HIGH TEMPERATURE QUENCHED QCD

SMALL (BARE) QUARK MASS,  $m/T = 0.24$

hot,  $T \approx 420$  MeV

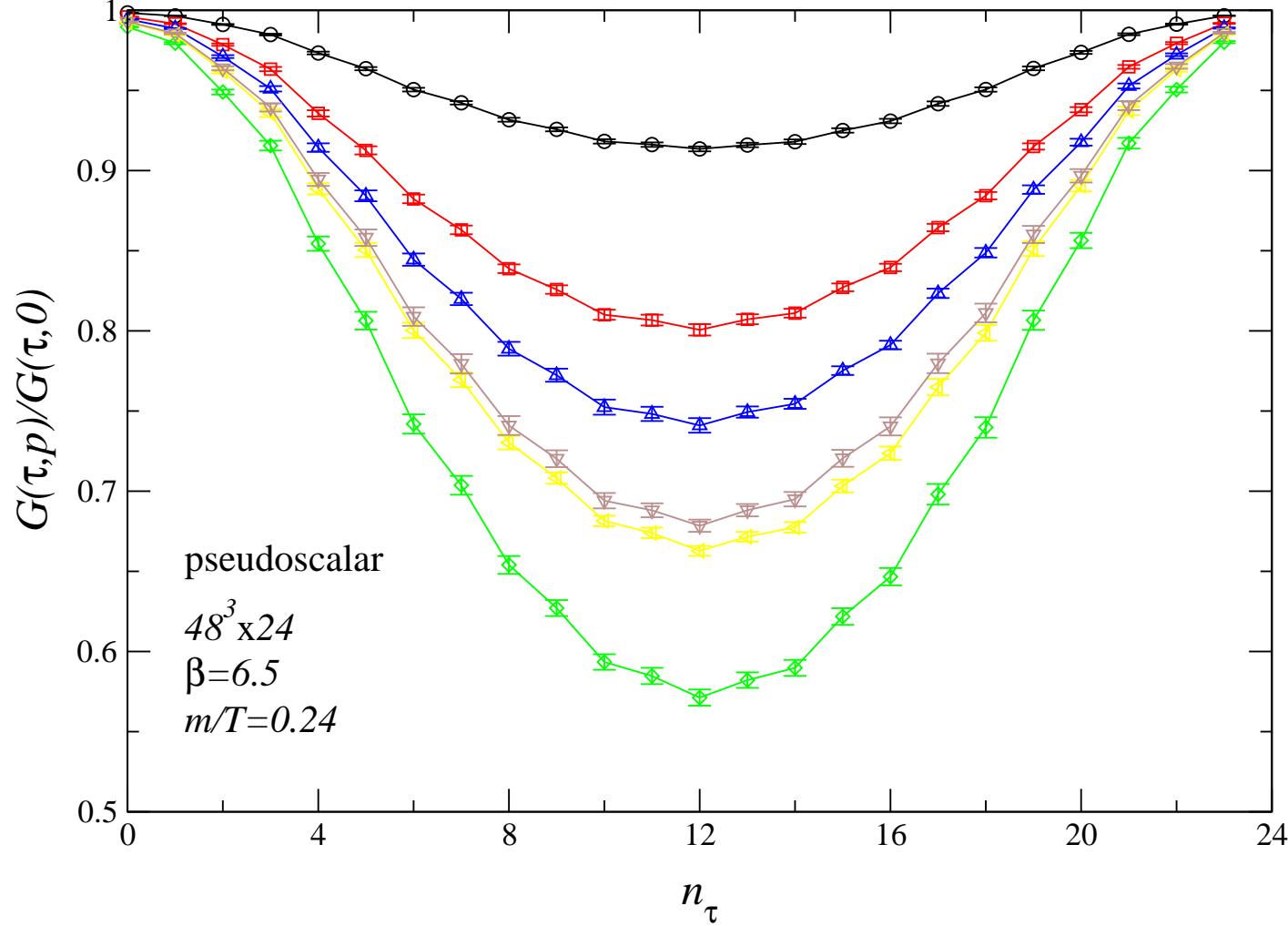


for comparison: free staggered quarks

# HIGH TEMPERATURE QUENCHED QCD

SMALL (BARE) QUARK MASS,  $m/T = 0.24$

cold,  $T \approx 160$  MeV



ratio in the confined phase: larger relative effect

# MAXIMAL ENTROPY METHOD (MEM)

BAYESIAN STATISTICS

solve ill-posed inversion problem

$$G_H(\tau, \mathbf{p}) = \int_0^\infty \frac{d\omega}{2\pi} K(\tau, \omega) \rho_H(\omega, \mathbf{p})$$

with the kernel  $K(\tau, \omega) = \cosh[\omega(\tau - 1/2T)] / \sinh(\omega/2T)$

$P[\rho|GH]$ : probability to find  $\rho$ , given  $G$  and taking into account prior information  $H$

write as  $P[\rho|GH] = P[G|\rho H]P[\rho|H]/P[G|H]$

$P[G|\rho H] \sim e^{-L}$  likelihood ( $\chi^2$  fit)

$P[\rho|H] \sim e^{\alpha S}$  prior probability, entropy

$$S = \int d\omega (\rho(\omega) - \rho_0(\omega) - \rho(\omega) \log [\rho(\omega)/\rho_0(\omega)])$$

extremize  $\exp(\alpha S - L)$

# STAGGERED FERMIONS

## STAGGERING EFFECT

spectral relation reads

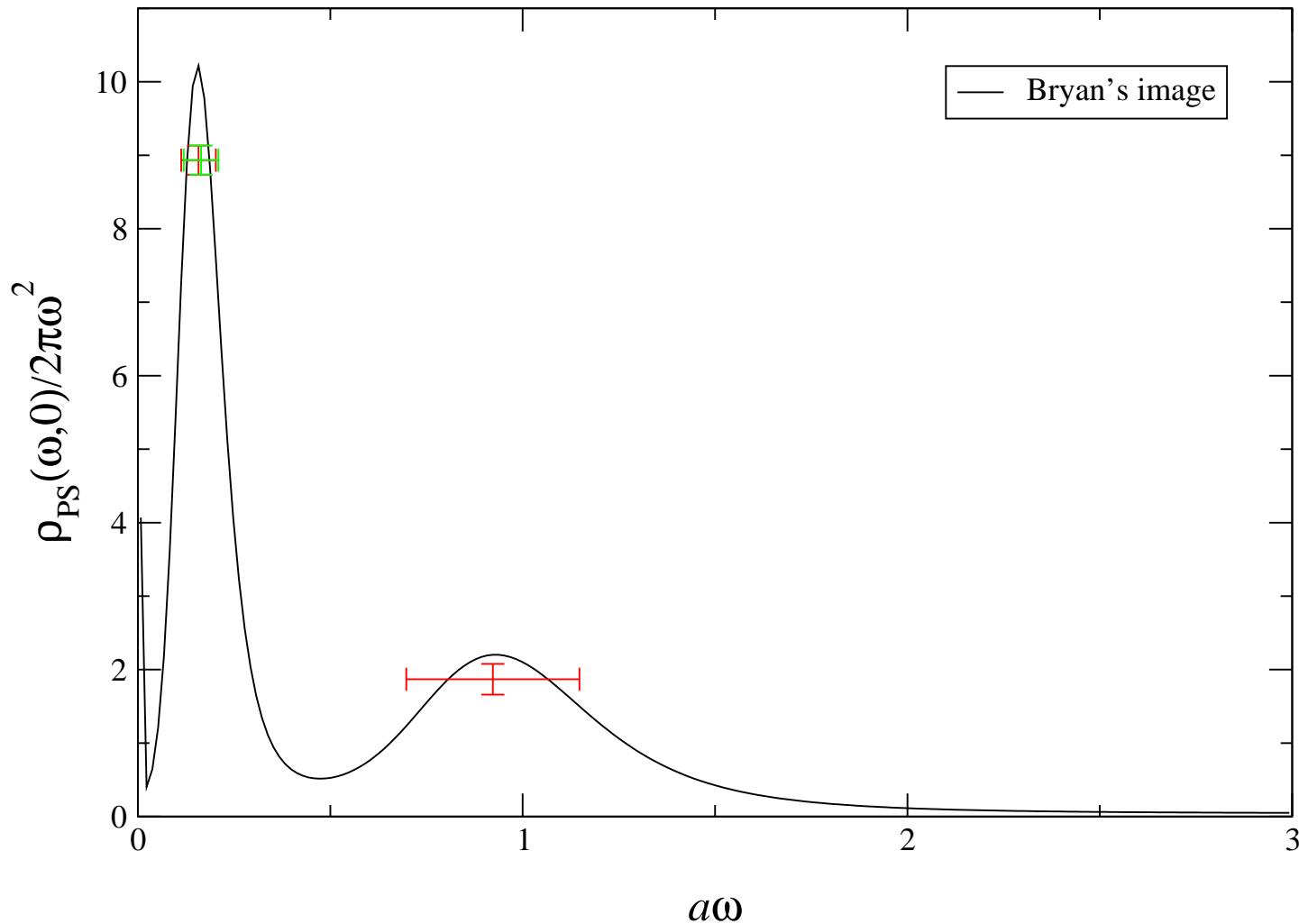
$$G_H(\tau, \mathbf{p}) = \int_0^\infty \frac{d\omega}{2\pi} K(\tau, \omega) \left[ \rho_H(\omega, \mathbf{p}) - (-1)^{\tau/a_\tau} \tilde{\rho}_H(\omega, \mathbf{p}) \right]$$

- $\rho_H(\omega, \mathbf{p})$  wanted
- staggered partner  $\tilde{\rho}_H(\omega, \mathbf{p})$ : related via  $\tilde{\Gamma}_H = \gamma_4 \gamma_5 \Gamma_H$   
“parity doubler”
- in MEM investigation: independent analysis on even/odd timeslices

first: results below  $T_c$

# SPECTRAL FUNCTIONS FROM MEM

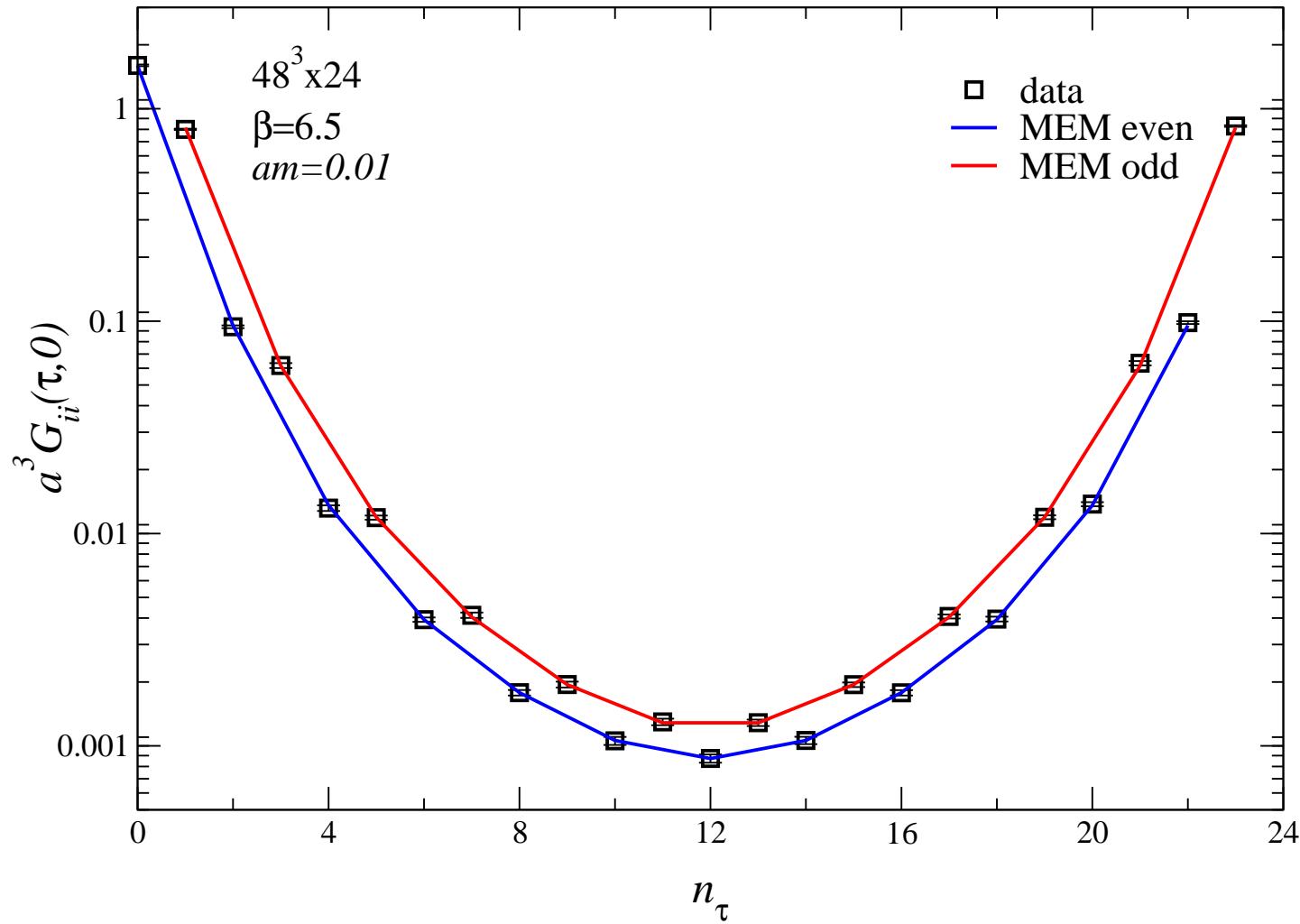
ACCURACY



pseudoscalar on even time slices  
error symbols: significance of the peaks

# SPECTRAL FUNCTIONS FROM MEM

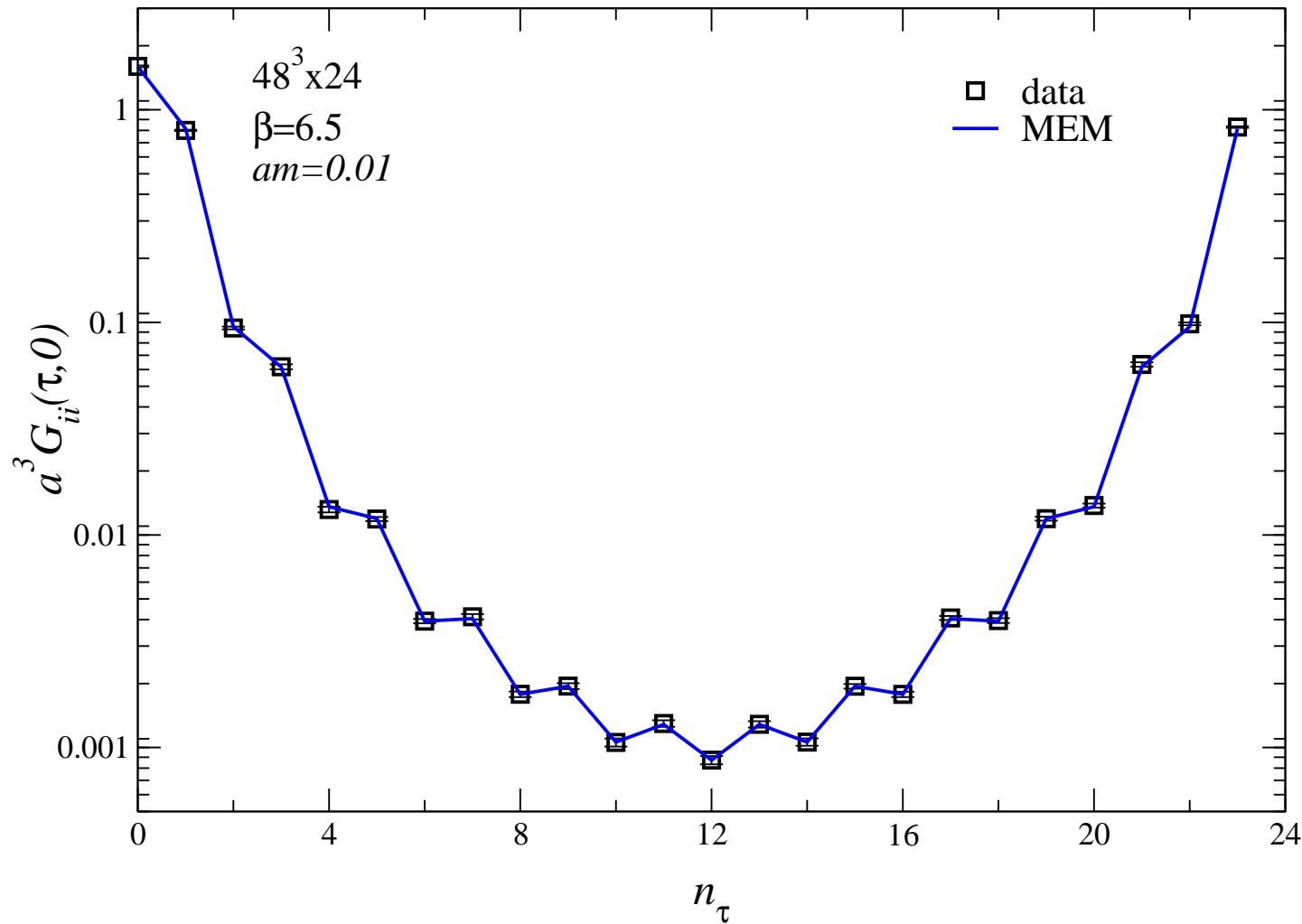
ACCURACY



staggering in the vector channel:  
independent MEM analysis to even and odd time slices

# SPECTRAL FUNCTIONS FROM MEM

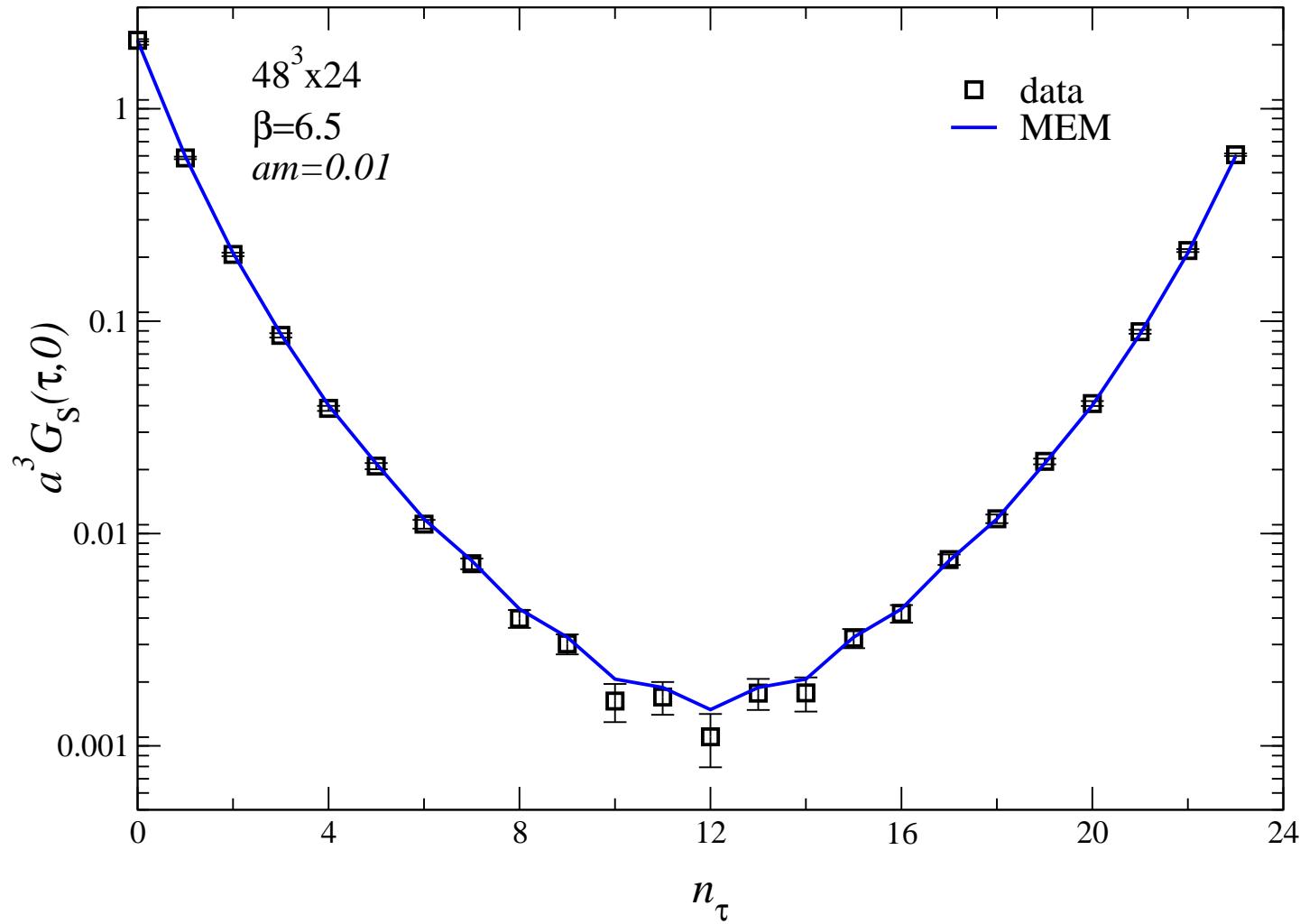
ACCURACY



staggering in the vector channel:  
combined result

# SPECTRAL FUNCTIONS FROM MEM

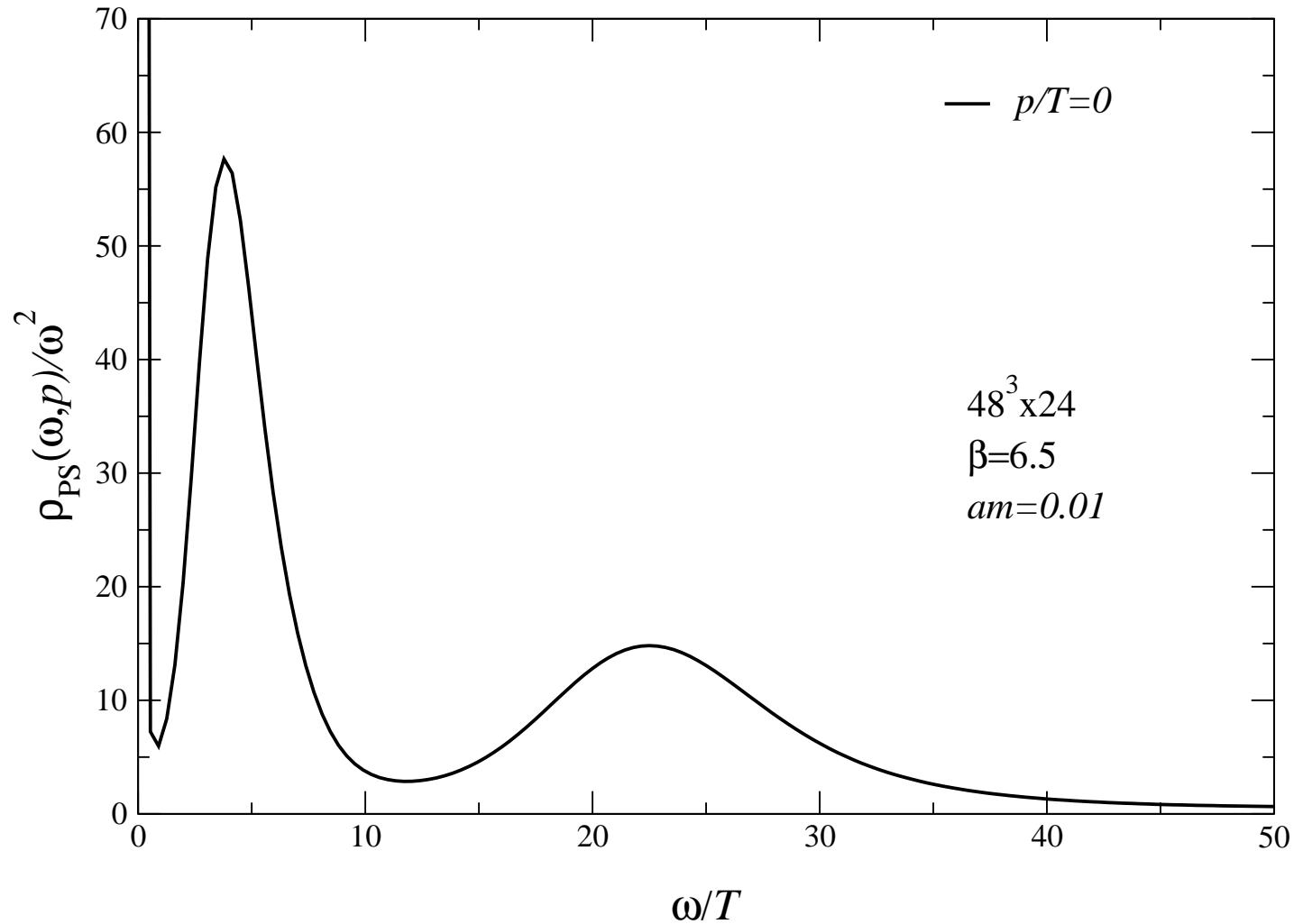
ACCURACY



scalar channel:  
effect of larger statistical uncertainty on MEM result

# MOMENTUM DEPENDENT SPECTRAL FUNCTIONS

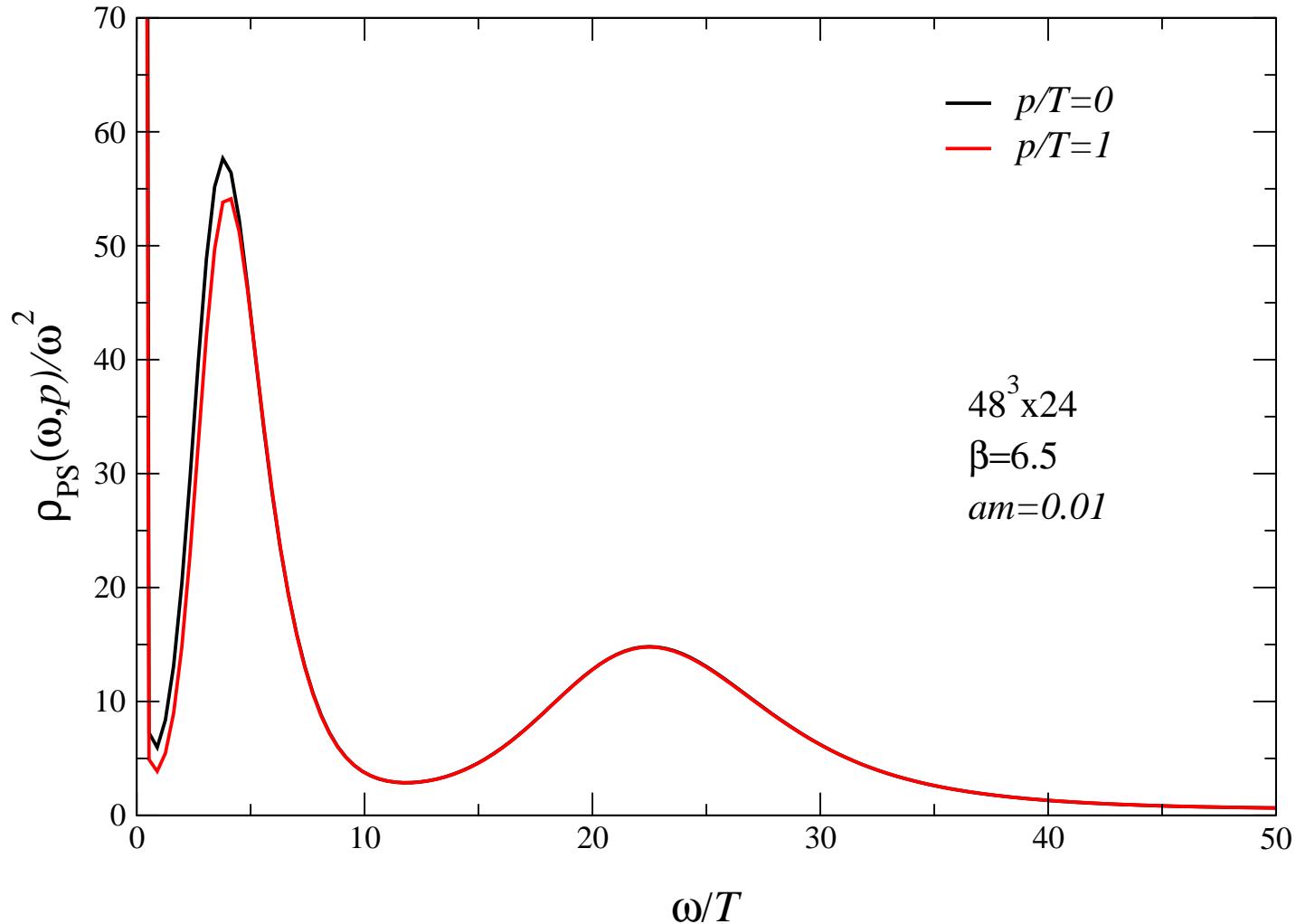
PSEUDOSCALAR, BELOW  $T_c$



zero momentum bound state

# MOMENTUM DEPENDENT SPECTRAL FUNCTIONS

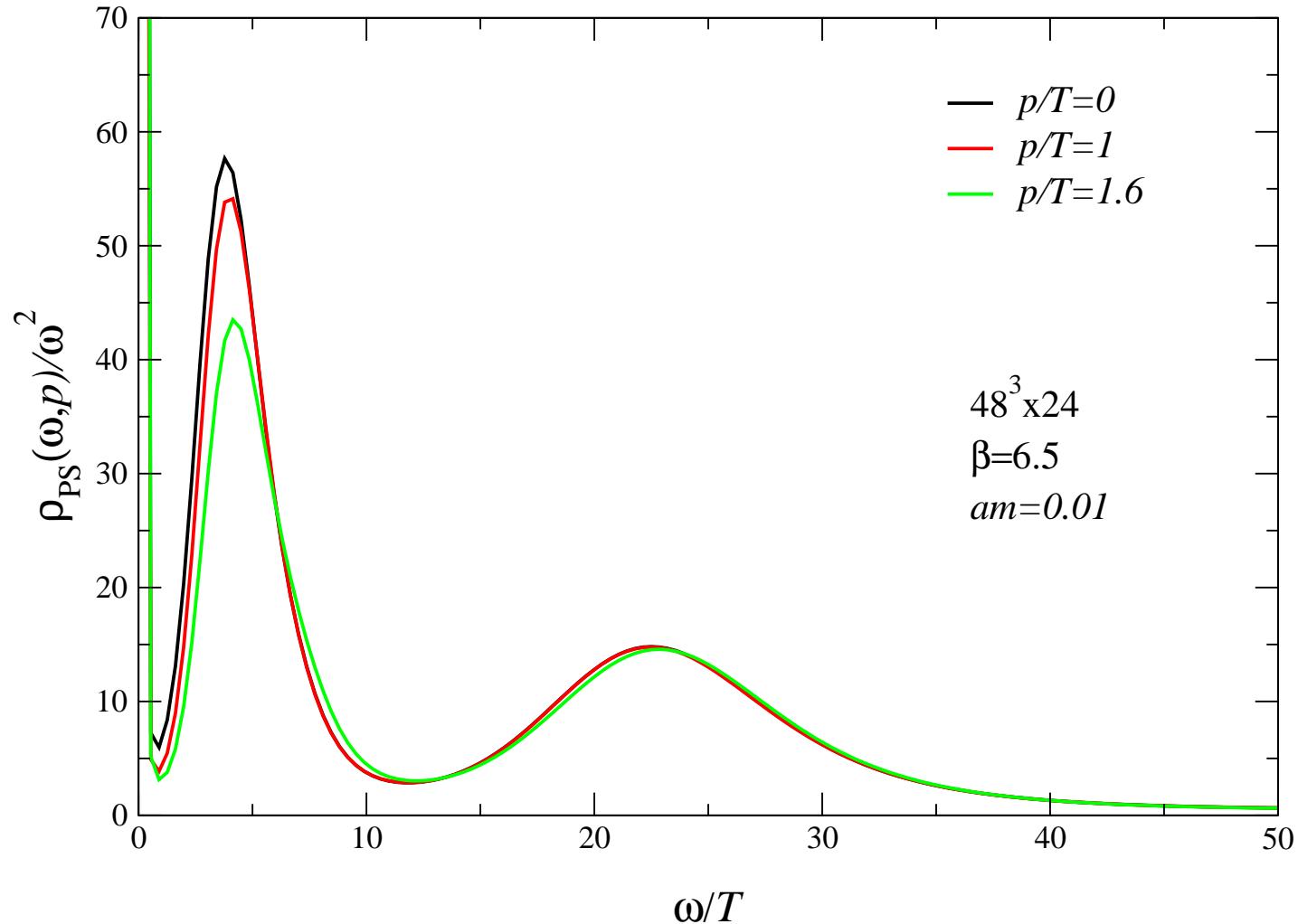
PSEUDOSCALAR, BELOW  $T_c$



increase momentum: moving quasiparticle

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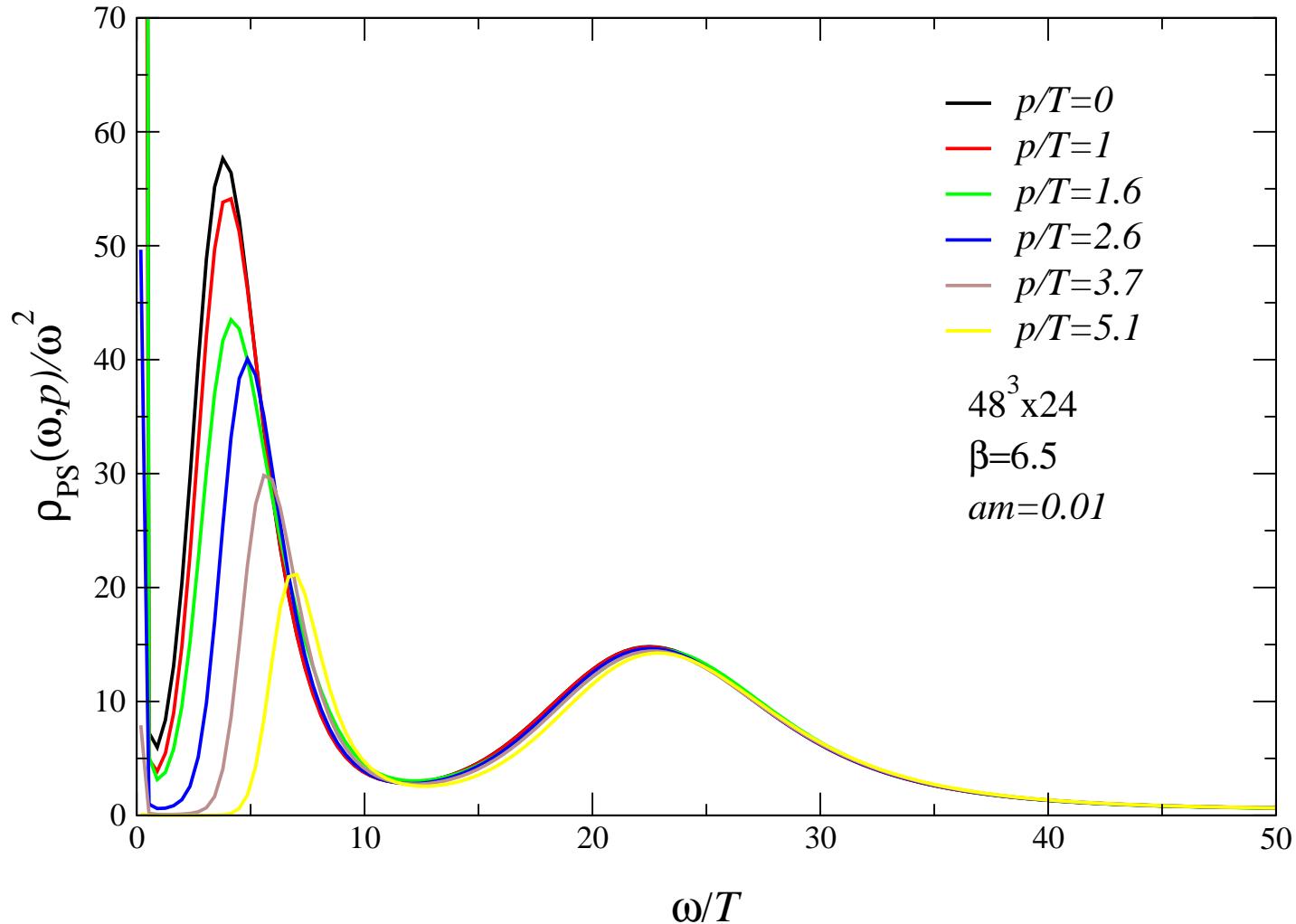
PSEUDOSCALAR, BELOW  $T_c$



increase momentum: moving quasiparticle

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PSEUDOSCALAR, BELOW  $T_c$



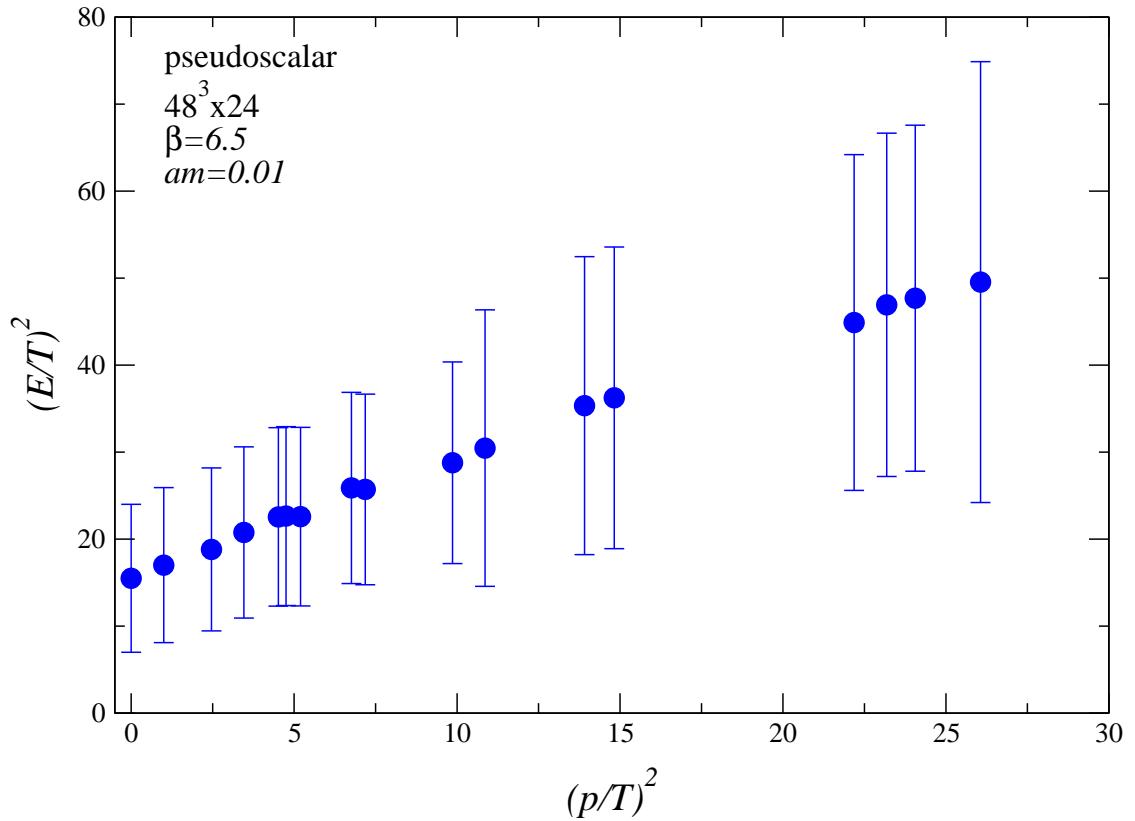
increase momentum: moving quasiparticle

# MOMENTUM DEPENDENT SPECTRAL FUNCTIONS

PSEUDOSCALAR, BELOW  $T_c$

moving quasiparticle:  
dispersion relation

error symbol =  
width of the  
spectral function

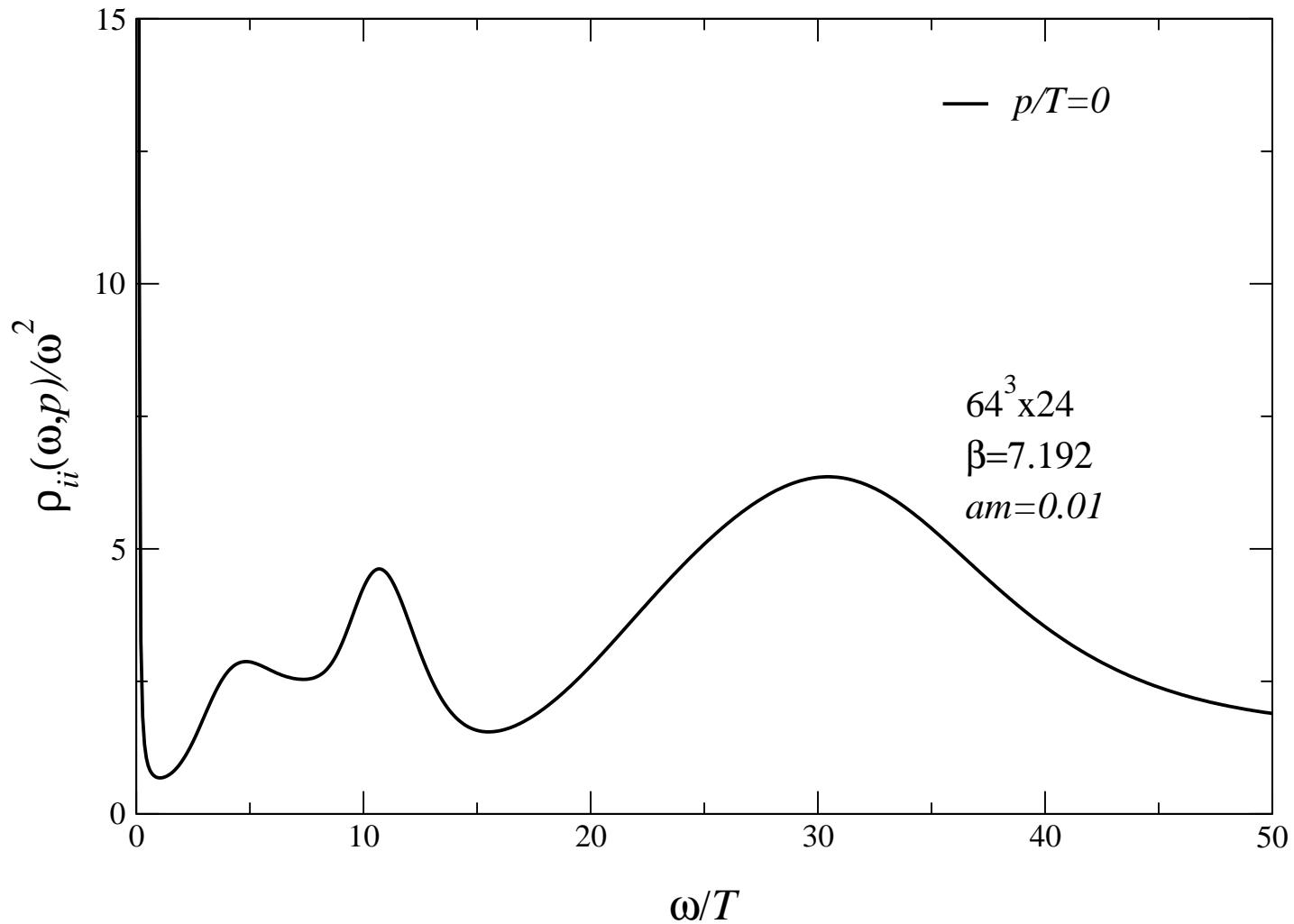


- is width physical or result of finite statistics?
- need more analysis

above  $T_c$ : vector channel

# MOMENTUM DEPENDENT SPECTRAL FUNCTIONS

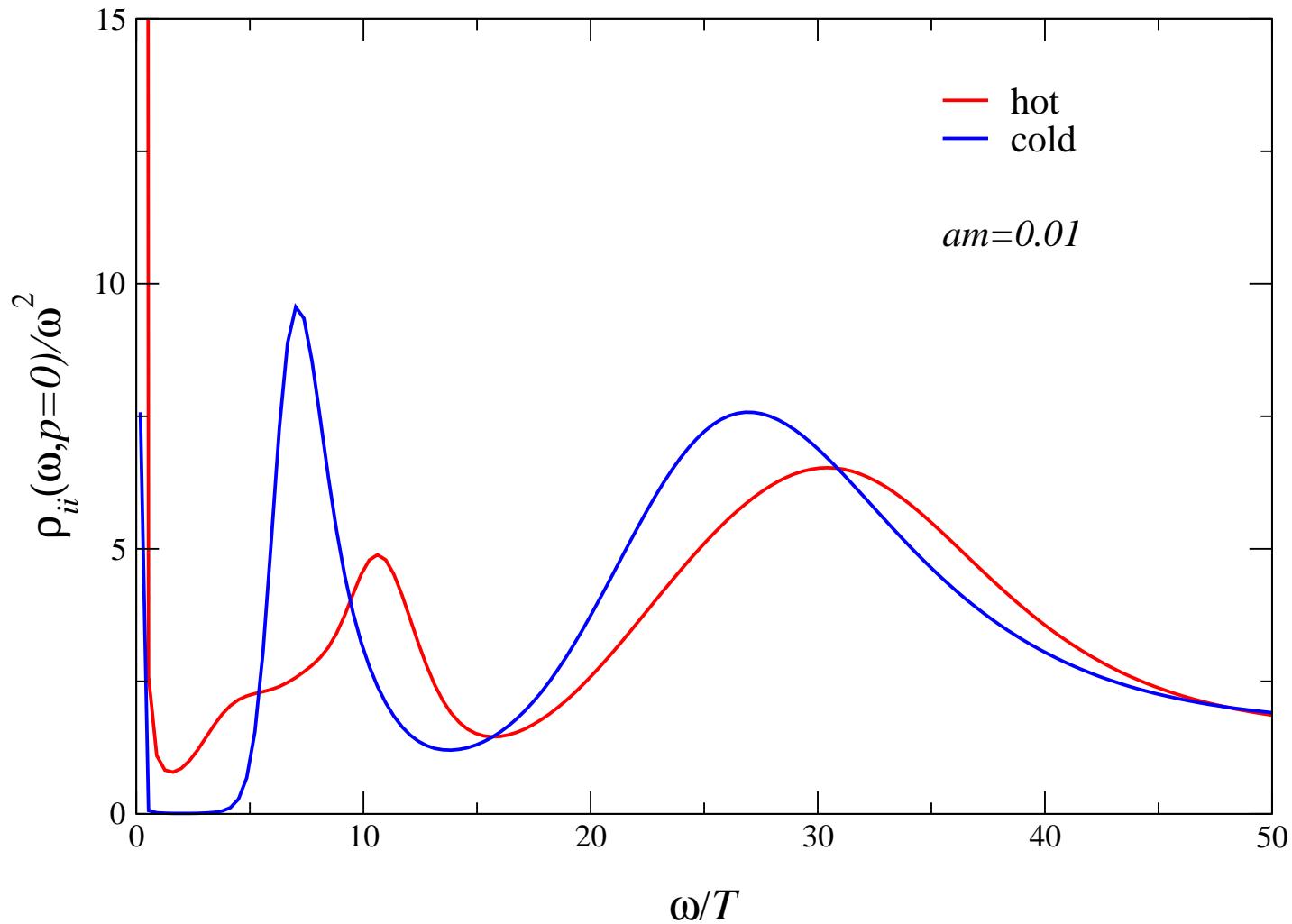
VECTOR CHANNEL, ABOVE  $T_c$



vector at zero momentum

# MOMENTUM DEPENDENT SPECTRAL FUNCTIONS

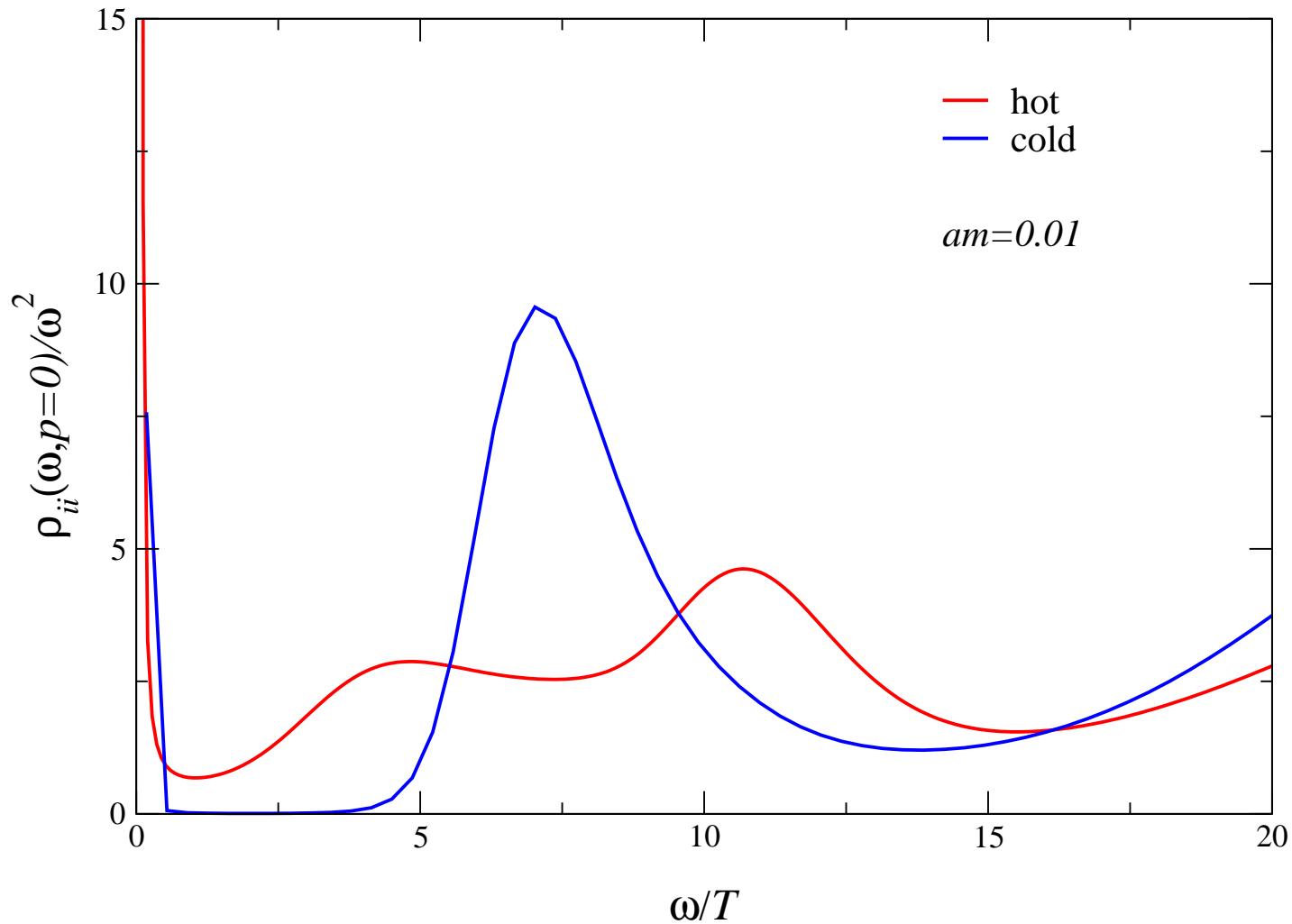
VECTOR CHANNEL, ABOVE  $T_c$



comparison below and above  $T_c$ : bound state is gone  
(note: different lattice spacing, physical quark mass)

# MOMENTUM DEPENDENT SPECTRAL FUNCTIONS

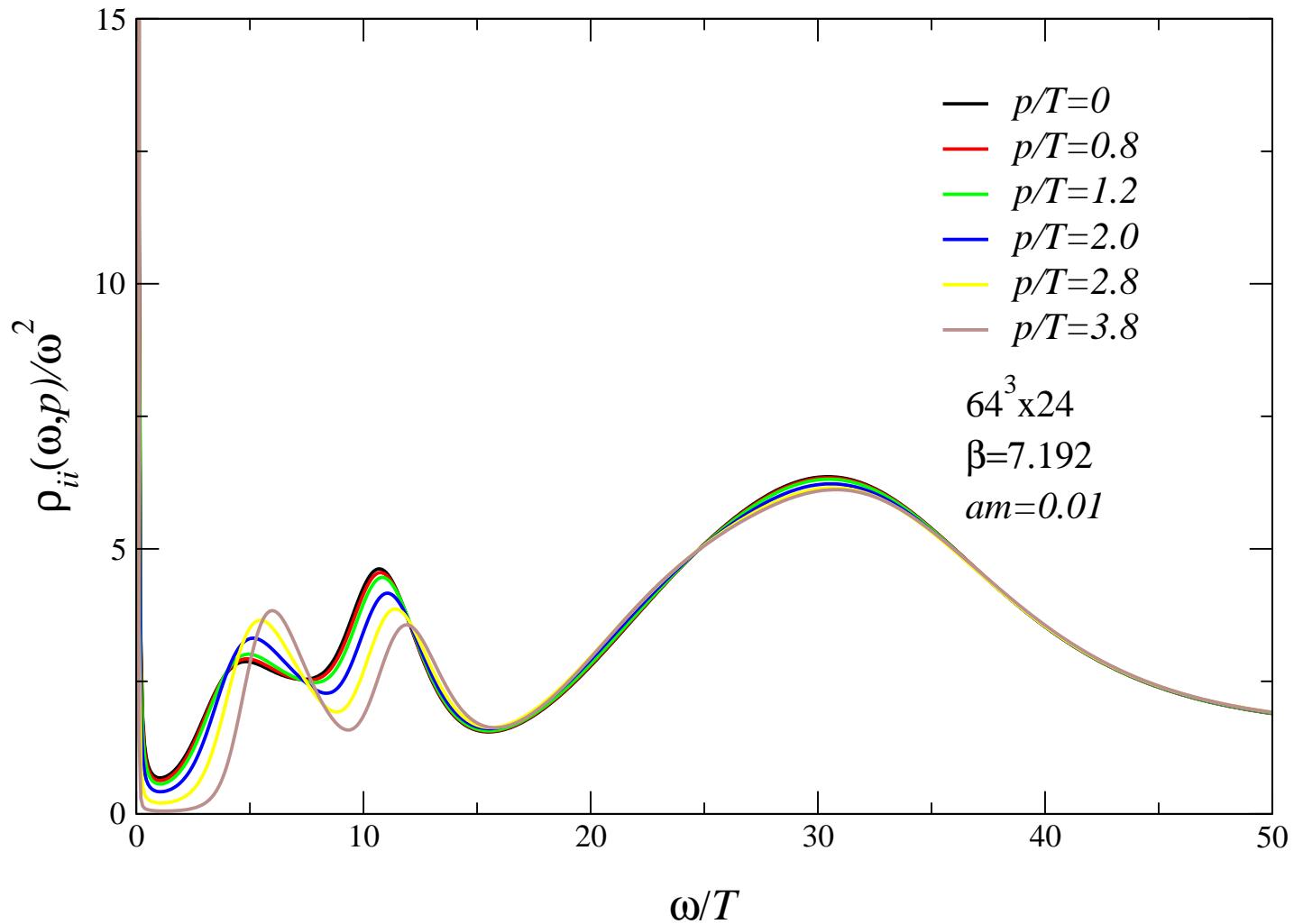
VECTOR CHANNEL, ABOVE  $T_c$



comparison below and above  $T_c$ : melting of the meson  
non-zero spectral weight at all  $\omega$

# MOMENTUM DEPENDENT SPECTRAL FUNCTIONS

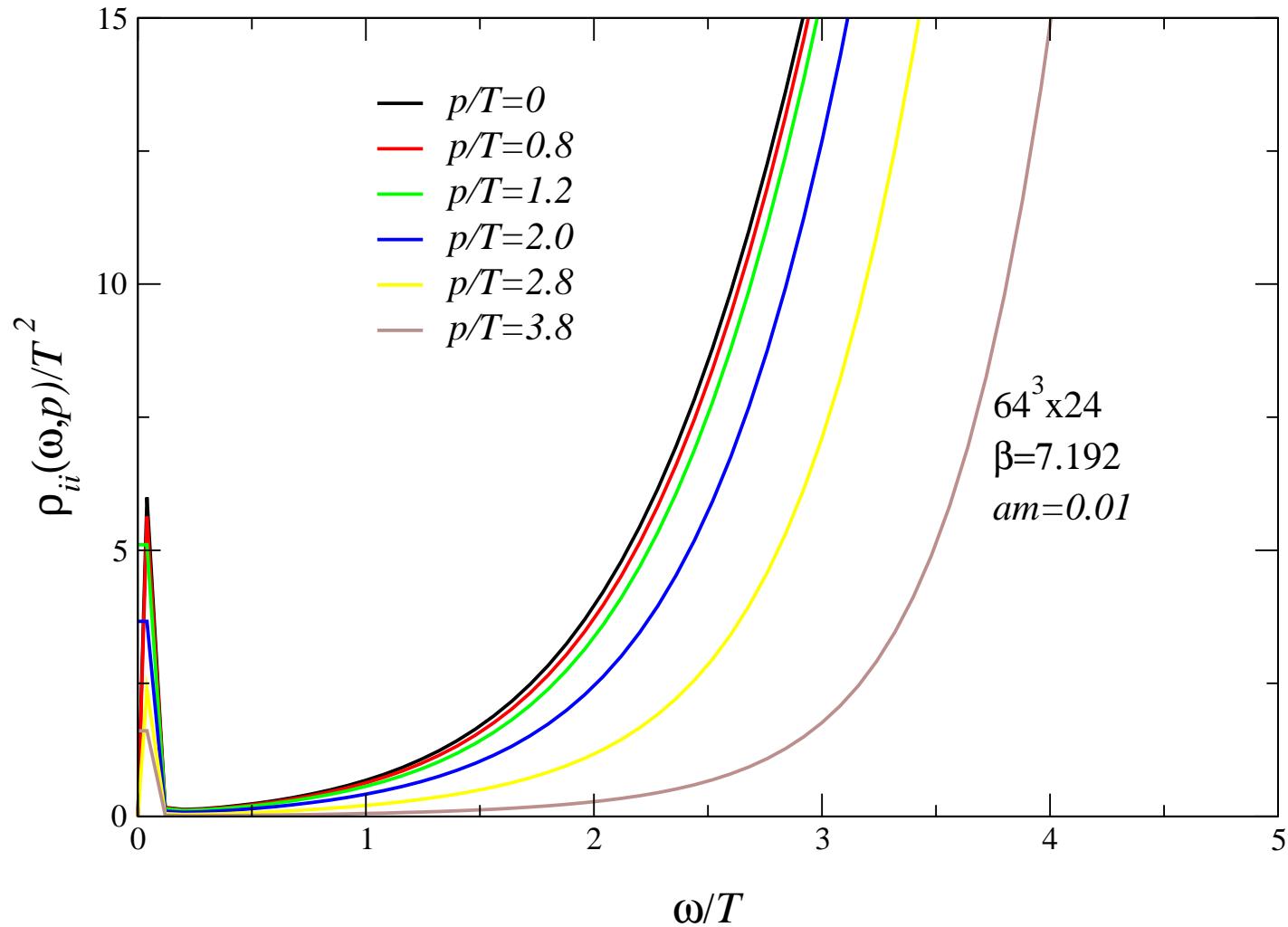
VECTOR CHANNEL, ABOVE  $T_c$



vector spectral function  $\rho^{ii}(\omega, p)/\omega^2$  at six different momenta

# MOMENTUM DEPENDENT SPECTRAL FUNCTIONS

VECTOR CHANNEL, ABOVE  $T_c$



vector spectral function  $\rho^{ii}(\omega, p)/T^2$   
at six different momenta: threshold (?)

# SUMMARY

- meson spectral functions at non-zero momentum
- non-trivial but ‘easier’ than transport coefficients
- light quarks: clear difference between below  $T_c$  and above  $T_c$
- first results from MEM below  $T_c$ 
  - moving bound states
- first results from MEM above  $T_c$ 
  - bound states melted
  - non-zero spectral weight everywhere
- all results preliminary: lots of work to be done